764

VOLUME 81 PAPER No. 764

PROCEEDINGS

AMERICAN SOCIETY
OF
CIVIL ENGINEERS

AUGUST, 1955

PLASTIC STRENGTH OF STEEL FRAMES

by Lynn S. Beedle, A.M. ASCE

STRUCTURAL DIVISION

{Discussion open until December 1, 1955}

Copyright 1955 by the American Society of Civil Engineers
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N. Y.

PRICE $0.50 PER COPY

THIS PAPER

--represents an effort by the Society to deliver technical data direct from the author to the reader with the greatest possible speed. To this end, it has had none of the usual editing required in more formal publication procedures.

Readers are invited to submit discussion applying to current papers. For this paper the final date on which a discussion should reach the Manager of Technical Publications appears on the front cover.

Those who are planning papers or discussions for "Proceedings" will expedite Division and Committee action measurably by first studying "Publication Procedure for Technical Papers" (Proceedings Paper No. 290). For free copies of this Paper—describing style, content, and format—address the Manager, Technical Publications, ASCE.

Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

The Society is not responsible for any statement made or opinion expressed in its publications.

This paper was published at 1745 S. State Street, Ann Arbor, Mich., by the American Society of Civil Engineers. Editorial and General Offices are at 33 West Thirty-ninth Street, New York 18, N. Y.

PLASTIC STRENGTH OF STEEL FRAMES

Lynn S. Beedle,[a] A.M. ASCE

SYNOPSIS

In order to demonstrate the applicability of plastic analysis to structural design in steel, the results of large-scale experiments are discussed in the light of theoretical predictions and correlated with current design specifications. The fundamentals of plastic analysis are briefly described. It is shown how the design of ordinary structural frames may be based on the maximum load the structure will support, as against the conventional methods which are based on the load at "first yield."

Although this report does not purport to be a complete review of current investigations into the behavior of steel structures loaded beyond the elastic limit, sufficient evidence is presented to show that in many instances design may properly be based upon ultimate strength.

INTRODUCTION

While the subject of design of steel frames on the basis of ultimate strength is not new, it is only in recent years that sufficient tests of large-size structural members and frames have been performed and adequate analytical techniques developed to make the topic of practical use to engineers in this country.

Curiously enough, studies such as those now under way at Lehigh University(8) and at other institutions, will enable the re-establishment of a technique that was accepted and used in Hungary as far back as the 1920's in the design of apartment buildings.(7) So, historically speaking, the topic is not new.

Many investigators have contributed prominently to the application of plastic analysis in structural design. Foremost in stimulating progress during the last ten to fifteen years has been J. F. Baker at Cambridge University, England.(3) Important contributions have also been made by Roderick, Horne, Symonds, and Neal.(10)

The purpose of this paper is to show examples of structures (approaching full size) that have been tested beyond the elastic and into the plastic range; to compare the maximum strength with the allowable load according to conventional elastic design; and to indicate how the reserve of strength beyond the elastic limit may be utilized in the design of ordinary structural frames.

What is plastic design? It will first be necessary to give some thought to the criteria by which the usefulness of a structure is judged. The structural strength of a steel frame may be determined by a number of factors, any one of which may actually constitute a "limit of structural usefulness." These possible design criteria are:

---

a. Asst. Director, Fritz Eng. Lab., Lehigh Univ., Bethlehem, Pa.

1) first attainment of a hypothetical yield point stress
2) brittle fracture
3) fatigue
4) instability
5) attainment of maximum plastic strength
6) large deflections.

Strictly speaking, a design based on any one of the six factors could be referred to as a "limit design." "PLASTIC DESIGN" as an aspect of limit design and as applied to continuous beams and frames embraces primarily Item 5—the attainment of maximum plastic strength.

Plastic design, then, is first a design on the basis of the maximum load the structure will carry (as determined from an analysis of strength in the plastic range). Secondly it consists of consideration by rules or formulas of certain "limitations" that might otherwise prevent the structure from attaining the computed maximum load. Many of these same limits may be present in conventional design (brittle fracture, fatigue). Others are inherently associated only with the plastic behavior of the structure. The unique feature of plastic design is that the ultimate load, rather than the yield stress, is regarded as the design criterion.

Why Plastic Design?

What is the justification for plastic design? One could reverse the question by asking, "Why use elastic design?". If the structure will support the load and otherwise meet its intended function, are the magnitudes of the stresses really important?

It is true that in simple structures the concept of the hypothetical yield point as a limit of usefulness is rational. This is because the ultimate load capacity of a simple beam is but 10 to 15% greater than the hypothetical yield point, and deflections start increasing very rapidly at such a load. While it would seem logical to extend elastic stress analysis to indeterminate structures, such procedures have tended to overemphasize the importance of stress rather than strength as a guide in engineering design and have introduced a complexity that now seems unnecessary for a large number of structures.

Actually the idea of design on the basis of ultimate load rather than allowable stress is a return to the realistic point of view that had to be adopted by our forefathers in a very crude way because they did not possess knowledge of mathematics and statics that would allow them to compute stresses.

The introduction of welding, of course, has been a very real stimulus to studies of the ultimate strength of frames. By welding it is possible to achieve complete continuity at joints and connections—and to do it economically. The full strength of one member may thus be transmitted to another.

It has often been demonstrated that elastic stress analysis cannot predict the real stress-distribution in a building frame with anything like the degree of accuracy that is assumed in the design. The work done in England by Prof. Baker and his associates as a forerunner to their ultimate strength studies clearly indicates this.(1)

Examples of "imperfections" that cause severe irregularity in measured stresses are: differences in beam-column connection fit-up and flexibility, spreading of supports, sinking of supports, residual stresses, flexibility assumed where actually there is rigidity (and vice-versa), and points of stress concentration. Such factors, however, usually do not influence the maximum plastic strength.

In summary, then, serious consideration is given to plastic design for three reasons:

1) Simplicity - most of the tedious analysis of the equations necessary for the elastic solution of indeterminate frames is eliminated. Also the "imperfections" mentioned above usually can be disregarded.

2) Rationality - by plastic analysis the engineer can determine with an accuracy that far exceeds his presently available techniques the real maximum strength of the structure. Thereby the factor of safety has more real meaning than at present. It is not unusual for the factor of safety to vary from 1.65 up to 3 or more for structures designed according to conventional methods.

3) Economy - because the maximum strength can be determined quite accurately, it is possible to utilize, with assured safety, the reserve of strength beyond the elastic limit.

Plastic design is not a technique that is intended to replace all design procedures. Structural design must preclude failure due to such things as brittle fracture, fatigue, and buckling—factors that may themselves become the design criterion. In ordinary building construction, such limitations are usually the exception, however, and not the rule. Therefore it can be expected that plastic design will find considerable application, particularly in continuous beams, industrial frames, and also in tier buildings.

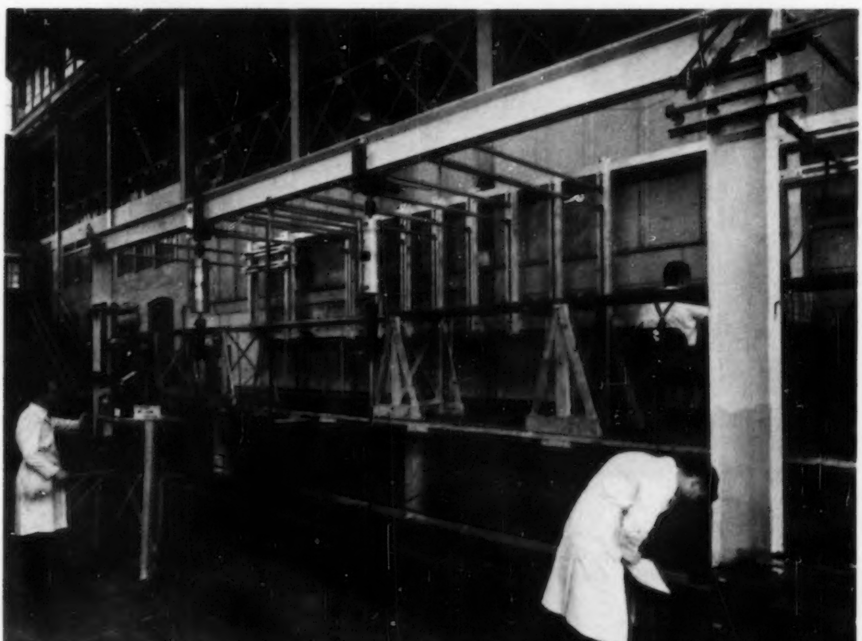


Fig. 1 General view of frame (T3) and Test Apparatus.

Fig. 2 Frame T3 after testing. Dotted line represents position of frame at maximum load.

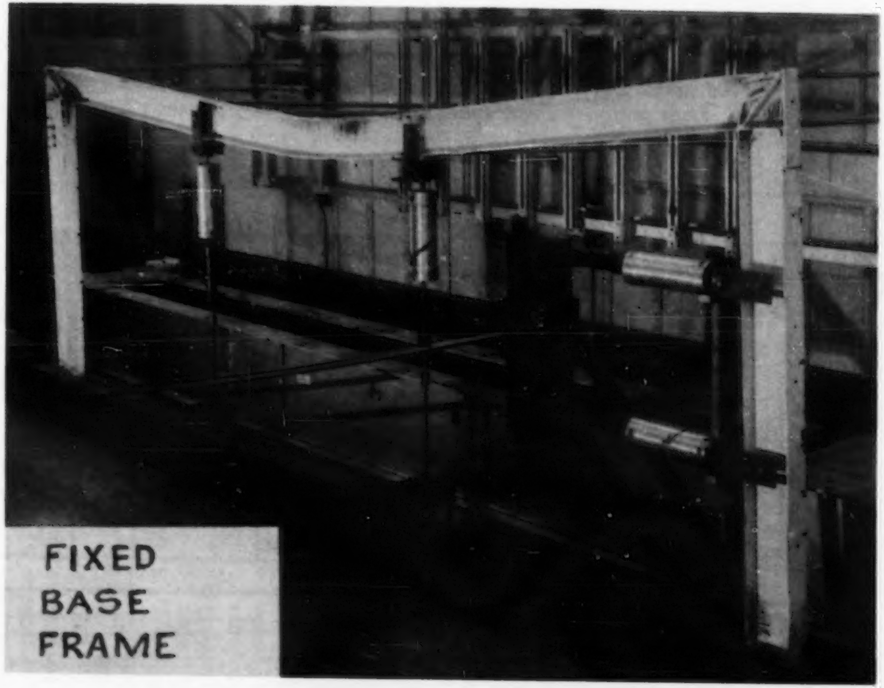


Fig. 3. Fixed-Base Frame (T4) at End of Test.

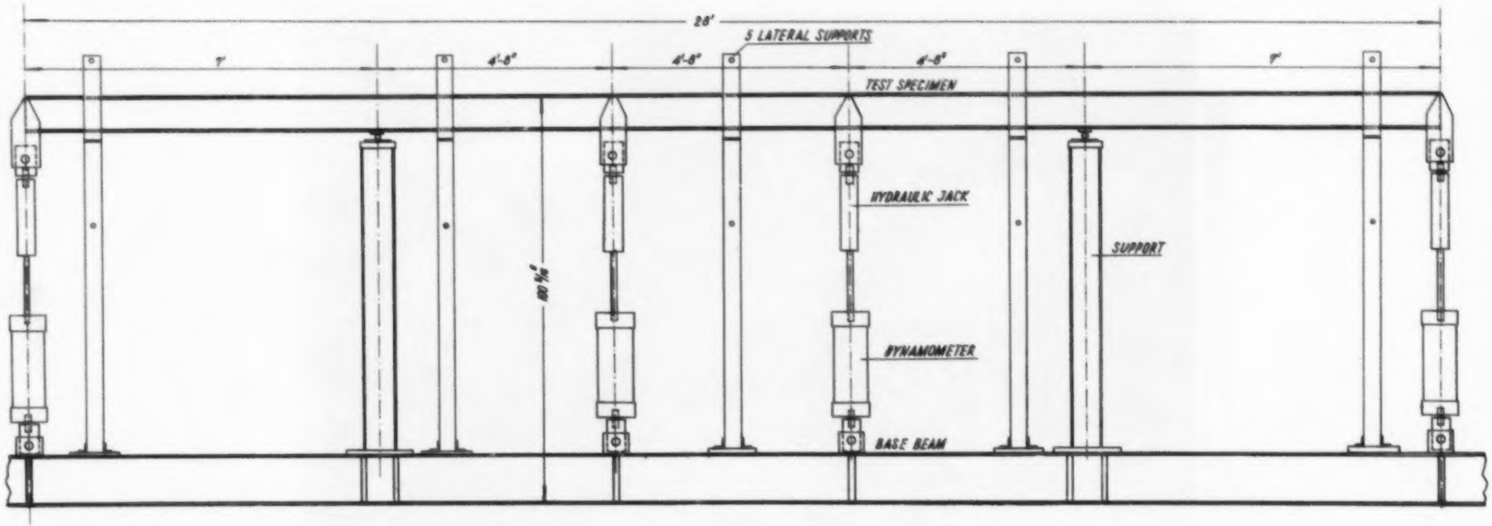


Fig. 4. Continuous Beam Test Set Up. Concentrated loads are applied at third points of central span and end loads are used to bring specimen to proper load condition.

Tests of Continuous Frames

The following pictures of continuous beams and frames show structures that were tested in a size approaching full-scale and will demonstrate the reserve strength of steel.

Fig. 1 shows the third in a series of four frame tests that have been completed at Lehigh University up to the present time. The span of the girder was 30 ft., the column height was 10 ft. The frame was pin-based and is shown in the early phase of the test. The vertical load was applied with hydraulic jacks at the third points of the beam. Side load, to simulate wind action was applied by the same technique. (The framing seen above the horizontal portion of the test frame is part of the Laboratory itself.)

In Fig. 2 the same frame is shown at the end of the test. In clearer detail are shown the arrangements for applying vertical and side load. With the solid line as a zero reference, the dotted line indicates the deflected shape when the frame first reached the predicted maximum load. The frame was deformed by jacking considerably beyond the maximum load, to demonstrate the toughness of steel frames strained beyond the elastic limit.

The bar chart at the right indicates the considerable reserve of strength above the elastic limit that is not utilized at present in structural design. The allowable load according to conventional elastic design is at the top of the clear portion of the chart. The predicted elastic limit is at the top of the cross-hatched portion, and the top of the bar is the maximum reserve strength—a value that is above the predicted ultimate load shown by the short horizontal line. Even with the extensive jacking, when the test was stopped (see arrow) the load was still above the maximum that could have been supported within the elastic limit.

The fourth frame tested in this series of single span frames is shown in Fig. 3. The column bases were fixed in this case, and this had the natural effect of decreasing the side-sway deflection. A second effect was to increase the ability of the frame to carry side load; the ratio of side load supported by this frame and that of the previous one was about 9 to 1.

The next few figures are of continuous beams tested in the program. Fig. 4 shows a typical set-up, designed in such a way that fixed-ended beams or beams with intermediate degrees of end restraint might be simulated. Loading is applied at the third-points. A fixed-ended beam is simulated by maintaining the supports in a level position through proper adjustment of the end loads. Typical details of connections are shown in Fig. 5. The beam in Fig. 6 was fabricated of a 14WF30 shape, continuous over two supports. The center span was 14 ft. with an overhang of 7 ft. at each end. The beam is shown at the beginning of the test. Fig. 7 is a closer view of one end.

In Fig. 8 is shown a picture of a portion of one of the continuous beams of 8WF40 shape. One of the load points is at the left and a support is near the middle of the picture. To the side is shown with a bar graph the maximum observed strength (105%), the computed maximum strength (100%), and the computed elastic limit (67%). There are also shown the allowable loads on the beam for 20,000 psi stress at the support (35%) and for the present AISC provision of 24,000 psi maximum stress at the support (42%). Such an increase of allowable stress at an interior support of a continuous beam is, of course, one means of utilizing some of the reserve of strength in design and was put into the AISC specifications in 1946.

It is seen from the previous pictures that the reserve strength of steel is considerable and that it can be predicted with reasonable certainty. Now, the question is, how can use be made of this strength in structural design.

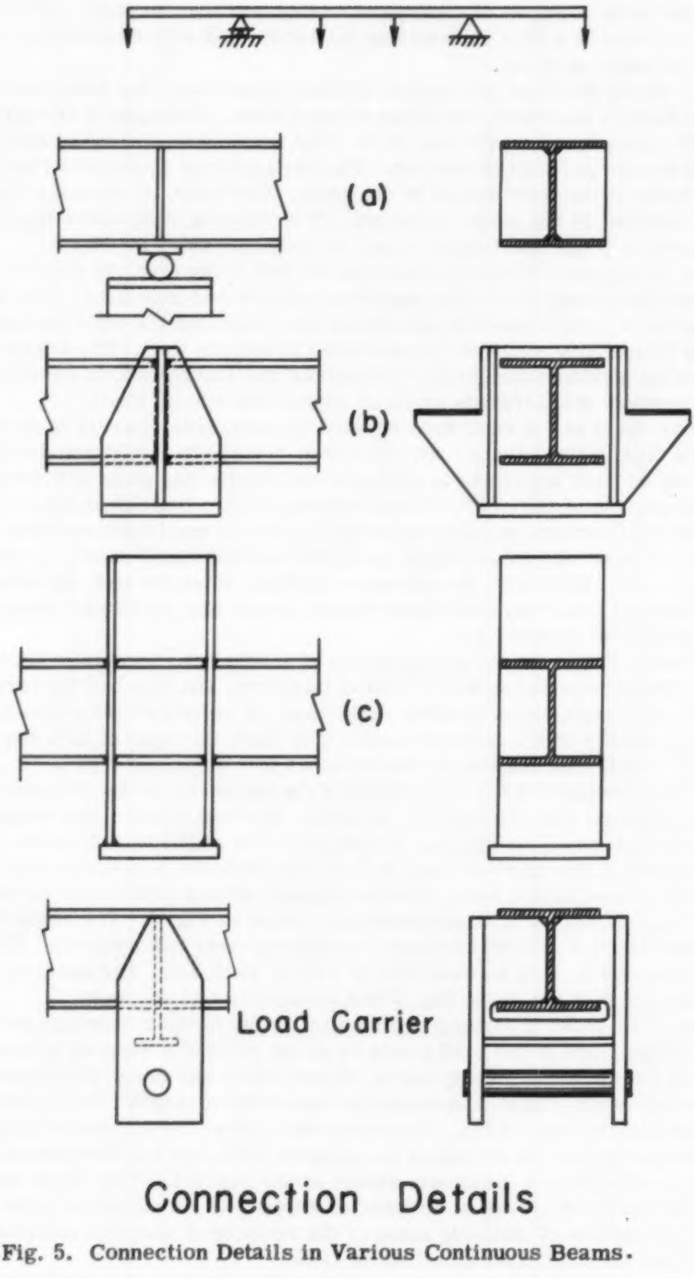


Fig. 5. Connection Details in Various Continuous Beams.

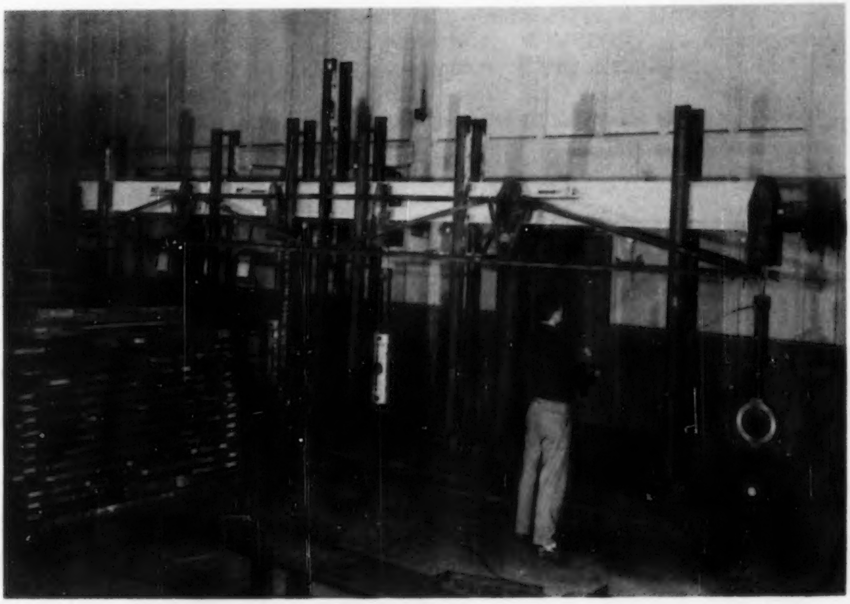


Fig. 6. Continuous Beam (B7) Under Test.

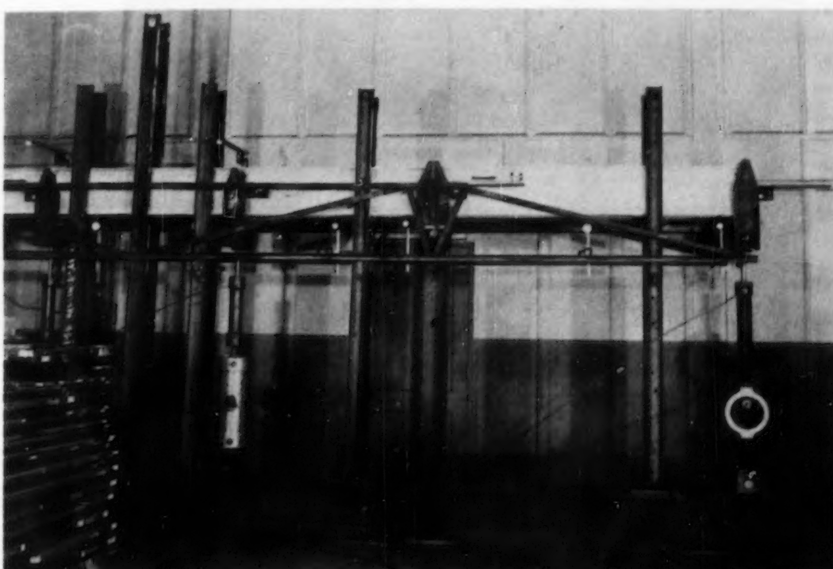


Fig. 7. West Portion of Continuous Beam (B7).

Fig. 8. Portion of Continuous Beam (B4) at End of Test; Support at Right, Load Point at Left. Shown as percentages of computed ultimate load (100%) are the observed maximum (105%), computed initial yield (67%) "allowable" -- 20,000 psi (35%), "allowable" -- 24,000 psi (42%).

The basis for computing this maximum plastic strength is the formation of "plastic hinges" at certain critical sections. The ability of columns, connections, and beams to form these all-important hinges is the key to the development of high reserve strengths.

Therefore, an examination will now be made of the basis of hinge formation; this will be followed by a study of components that have developed plastic hinges.

The Development of Reserve Strength

5.1 Plastic Hinges

A typical example of the familiar stress vs strain curve of ordinary structural steel is shown in Fig. 9. After the yield point is reached, a tensile specimen would stretch plastically without an increase in load for a distance that is from 10 to 15 times the extension at the elastic limit. This is followed by strain hardening, and at about 10 to 15% elongation, the curve reaches the maximum point.

Conventional elastic design makes use of only an insignificant portion of the strain capacity of steel—up to the first dotted mark or about a tenth of one percent elongation. Even in plastic analysis, at ultimate load the critical strains will not have exceeded about 1.5% elongation. Thus, the use of ultimate strength as the design criterion still leaves available a major portion of the reserve ductility of steel which can be used as an additional margin of safety.

The inset in Fig. 9 shows the first portion of the stress-strain curve, drawn to larger scale. By keeping in mind this "idealized" curve, it is possible to follow the formation of these plastic hinges.

In Fig. 10 bending moment is plotted against the rotation of a section bent by couples. This couple deforms the beam, introducing a certain unit rotation angle per unit length along the beam. If one imagines that all of the material in the beam is concentrated in its flanges, then the plot of moment vs curvature is just the same as the stress vs strain curve in Fig. 9—the compression flange acts like a compression specimen and the tension flange like a tension specimen. The analogy between an "actual hinge" and a "plastic hinge" is also shown in Fig. 10.

Fig. 11 shows a more realistic picture of the development of a plastic hinge and the successive change from elastic to plastic action. Point 1 is the elastic limit. At Point 2 the member is partially plastic, and Point 3 approaches the limiting "plastic hinge moment," M_p. As is evident from the stress-distribution, M_p may be computed from the yield stress and statical moment,

$$M_p = \sigma_y Z \tag{1}$$

where

σy = lower yield-point stress
Z = plastic modulus, computed as twice the static moment of half the cross-section

Z is analogous to S, the elastic modulus.

One of the two sources of reserve strength beyond first yield is also shown in Fig. 11. The development of plastic yield of the full cross-section results in an increase of bending strength beyond computed yield of about 14% for

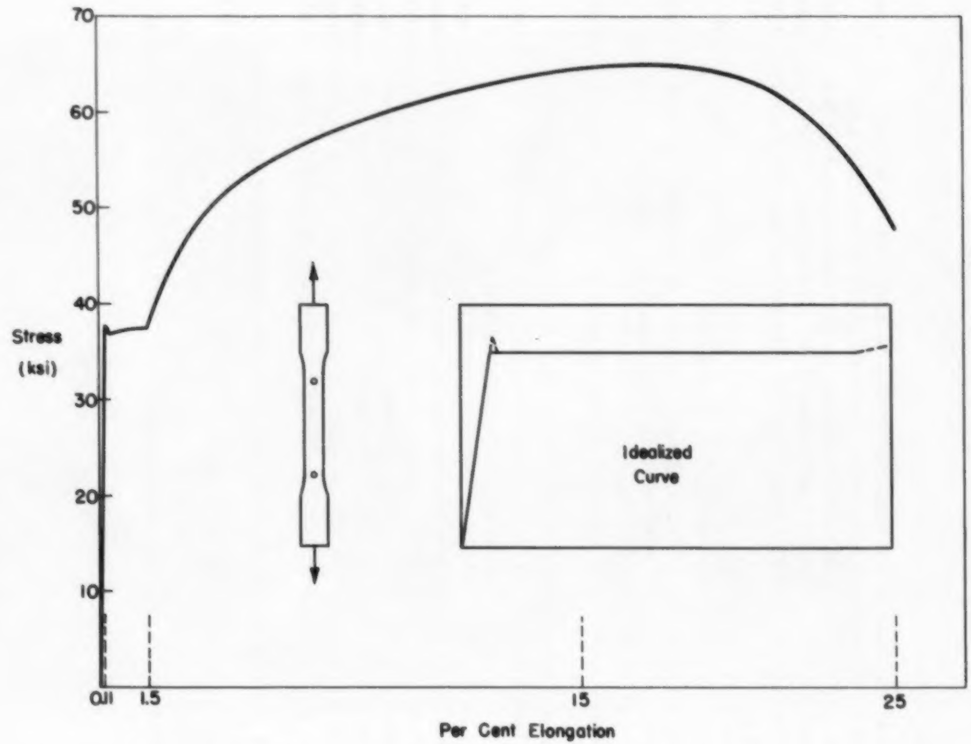


Fig. 9. Typical Stress-Strain Diagram for Structural Steel. Insert shows idealization of first portion of curve (up to 1.5% strain).

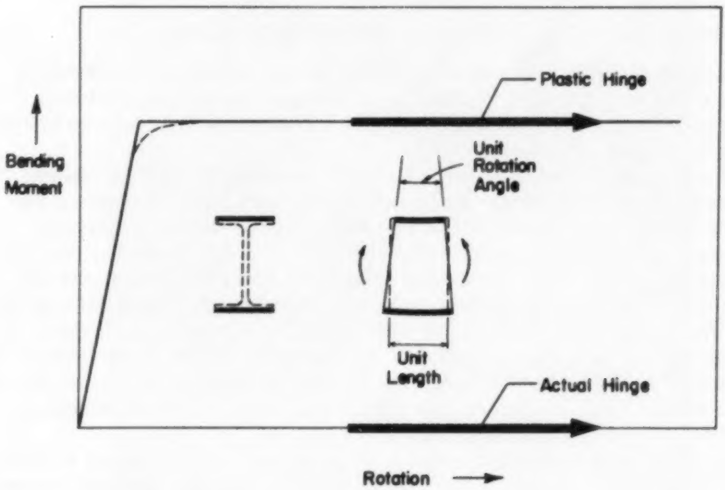


Fig. 10. Idealized M-ϕ Curve.

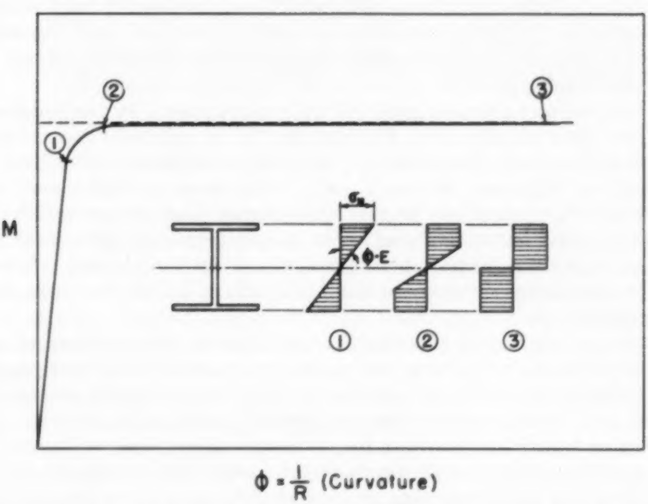


Fig. 11. Moment-Curvature (M-ϕ) Relationship for WF-Shape According to the Simple Plastic Theory. Corresponding stress distributions are shown for various points on the curve.

rolled WF shapes. The term "shape factor" denotes the ratio between the full strength (M_p) and first yield (M_y).

5.2 The Formation of Plastic Hinges in Structural Elements

A considerable amount of the work on the Lehigh program has involved testing of component parts of frames to see whether or not they actually develop the predicted plastic moment strength. There now follow some photographs of these component parts.

In Fig. 12 is shown one of the welded corner connections. The coating of whitewash was used to show the flaking of mill scale. In the corner of the connection it will be noticed that the web has yielded rather extensively, transferring load to the diagonal stiffener that was inserted for the purpose of carrying that load. This connection joined two 12WF36 shapes, and the moments were such that the joint was being "closed" under the action of the forces. The lower portion is the top of a column and it, too, participates to a considerable extent in the plastic action. The influence of direct stress is evident in the column since the inner flange (where bending and direct forces are additive) is yielded more than on the tension side. The neutral axis is thus shifted to the right of center by a small amount.

In Fig. 13 is shown a connection of similar design but fabricated to join two 36WF230 beams. The behavior is similar to that of the smaller shapes. In the lower portion of the figure is shown the non-dimensional curve of moment vs deflection, the latter being measured between the two arms of the connection. In the upper portion are photographs of the knee taken at three different stages of the test and as indicated on the lower test curve. At computed yield ("a") there is negligible deformation; as the maximum moment is approached ("b") modest plastic yield zones have developed; only after the connection has been rotated beyond any practical overload does the deformation become severe ("c"). "Unloading" is due to the combined effects of local and lateral buckling.

Plastic hinges may form at load points or at supports in continuous beams. Examples are shown in Fig. 14. Photograph "a" is a detail taken at the support as indicated in the upper sketch. Although somewhat obliterated by the auxiliary loading stiffener, the detail here is the same as that shown in Fig. 5b. Photograph "b" shows the region between two load points. (The loading is such that the moment is uniform in the center portion). The yield patterns differ depending on the magnitude of shear force that is present. However, studies of the influence of shear on the plastic have shown that it is not a severe limitation.

Plastic hinges may form in a continuous frame at connections, at points of concentrated load or reaction, and also at certain critical sections in beams where the bending moment is a maximum. Fig. 15 is typical of what might be expected for a beam in which the moment is practically uniform—such as at the center of a uniformly loaded span. Hinges form, and as expected, they extend over a somewhat longer length of the beam. Fig. 15 shows two views. At the top the yield point has been passed and the moment is approaching M_p. At the bottom is a view of the beam after the curvature had far exceeded what would be required to satisfy the conditions of plastic analysis. The severe deformation seen in the various photographs is accomplished in order to obtain a complete load history. The deformation at maximum load is, of course, much less.

Moment vs rotation curves demonstrating the capacity of structural components to form hinges are shown in Figs. 16 and 17. They correspond with

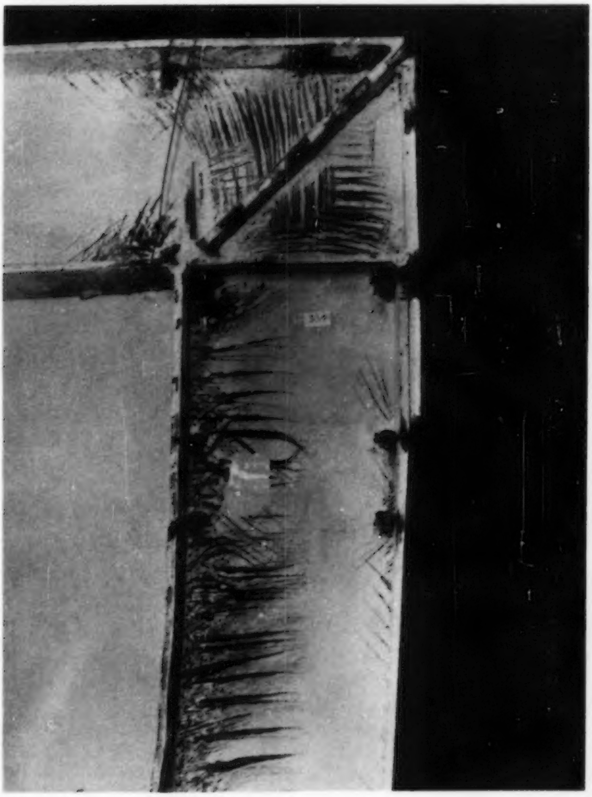


Fig. 12 Typical Behavior of Corner Connection Joining 12WF36 Shapes.

theoretical curves of Figs. 10 and 11. Fig. 16 presents a composite curve of 12 different corner connections. It includes the results of tests on 8-, 12-, 14-, 24-, and 36-in. wide-flange shapes. (An asterisk marks the curve for the 36-in. connection of Fig. 13.) In the figure, bending moment is plotted vertically against rotation horizontally, all coordinates having been reduced to the same common percentage basis. Although there are some differences in behavior, all of the connections developed a bending moment that is greater than the predicted plastic moment. Further, this plastic moment is maintained through a considerable rotation angle.

In Fig. 17 are shown moment-vs-rotation curves obtained from measurements made in the vicinity of the support of three continuous beams of 8WF40 shape. Sometimes the section develops a hinge moment that is a few percent less than the predicted plastic moment in the early phase of the test. However, with adequate lateral support and continued straining, the strength eventually reaches and usually exceeds the computed plastic hinge moment.

Behavior of other components is similar; thus we can depend on properly proportioned structural components to develop the plastic hinge moment and to have an adequate reserve of curvature.

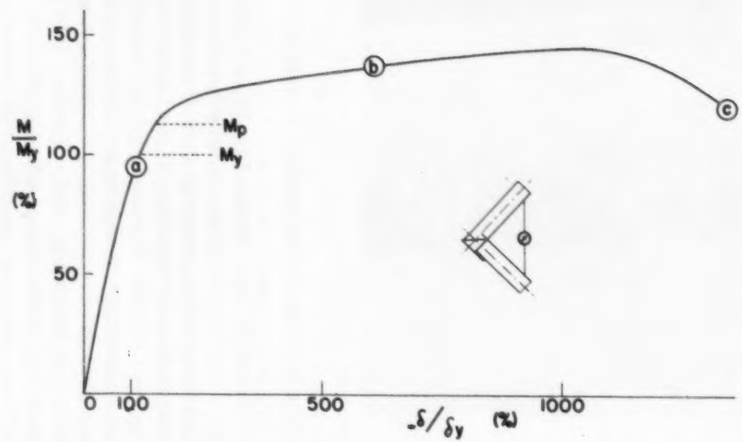


Fig. 13. Behavior of 36WF230 Corner Connection. Photographs show connection at first yield (a), near maximum (b), and following "unloading" (c) due to local and lateral buckling.

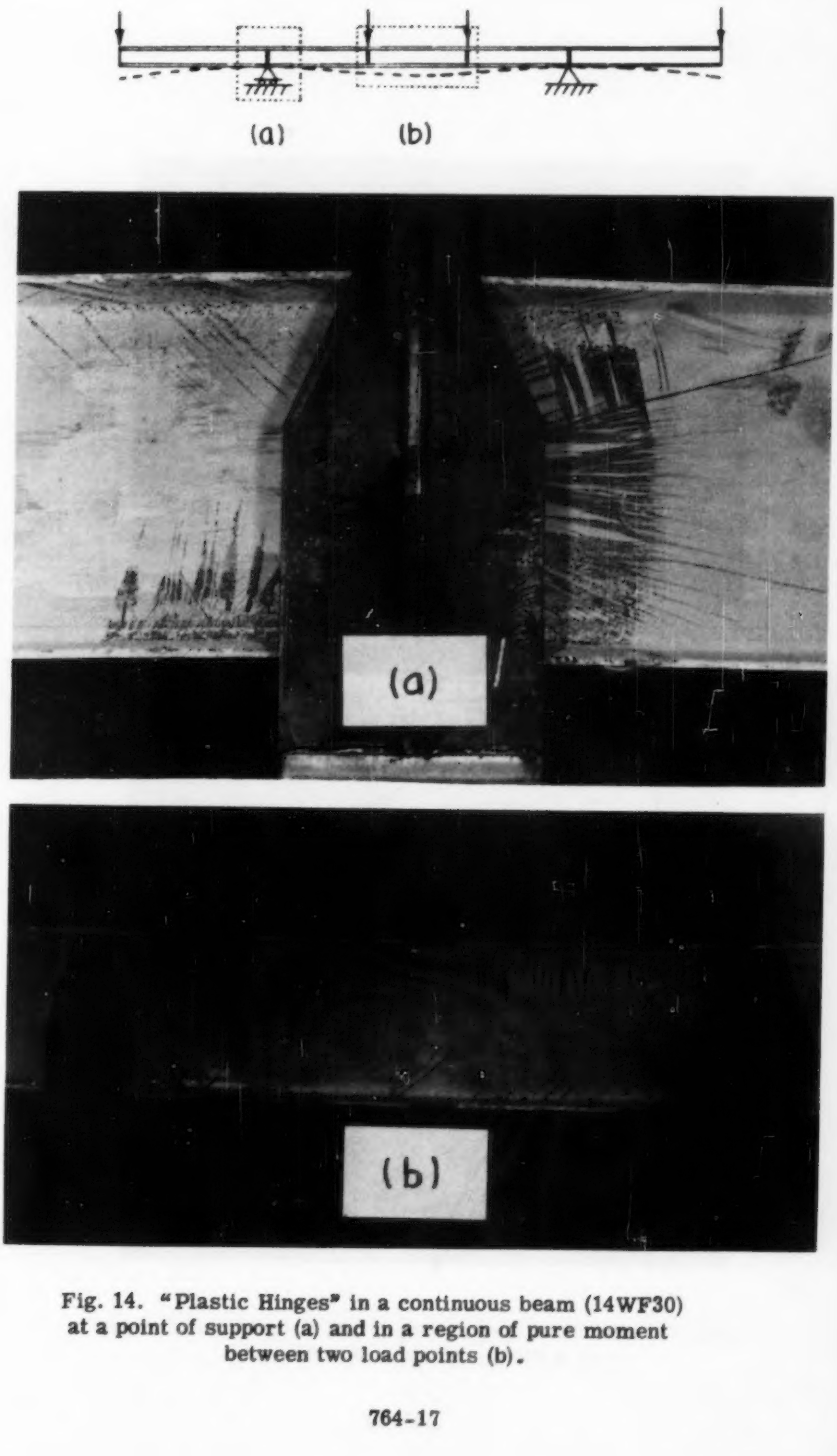


Fig. 14. "Plastic Hinges" in a continuous beam (14WF30) at a point of support (a) and in a region of pure moment between two load points (b).

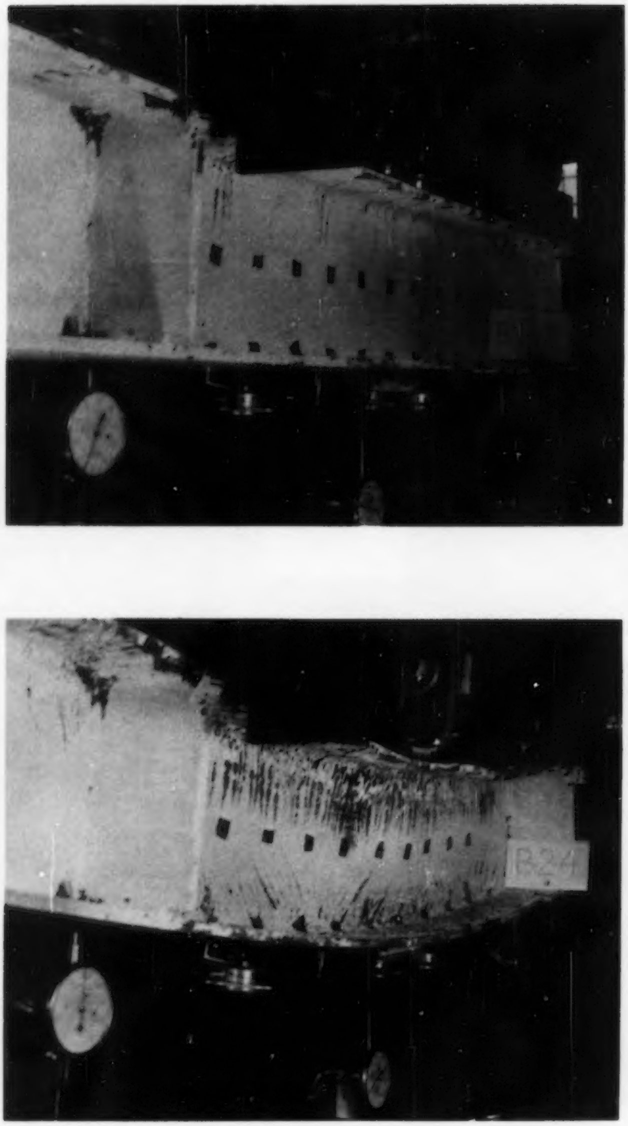


Fig. 15. The Development (upper and Progression (lower) of a Plastic Hinge in WF Beam.

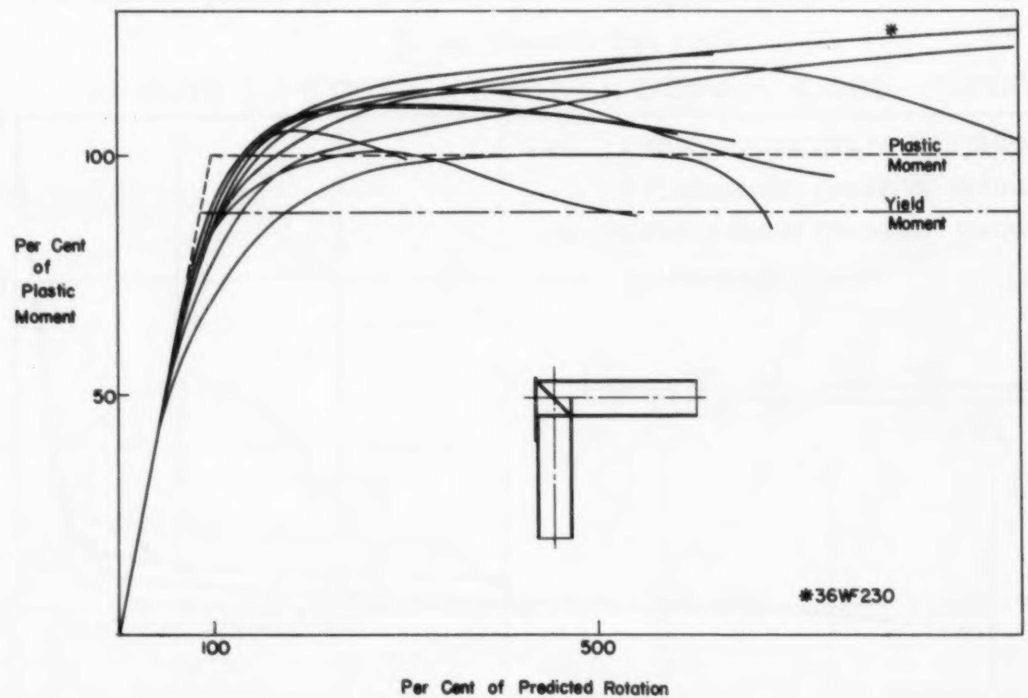


Fig. 16. Composite moment-rotation curves for various corner connections illustrating the attainment of plastic moment and further rotation at near-constant moment.

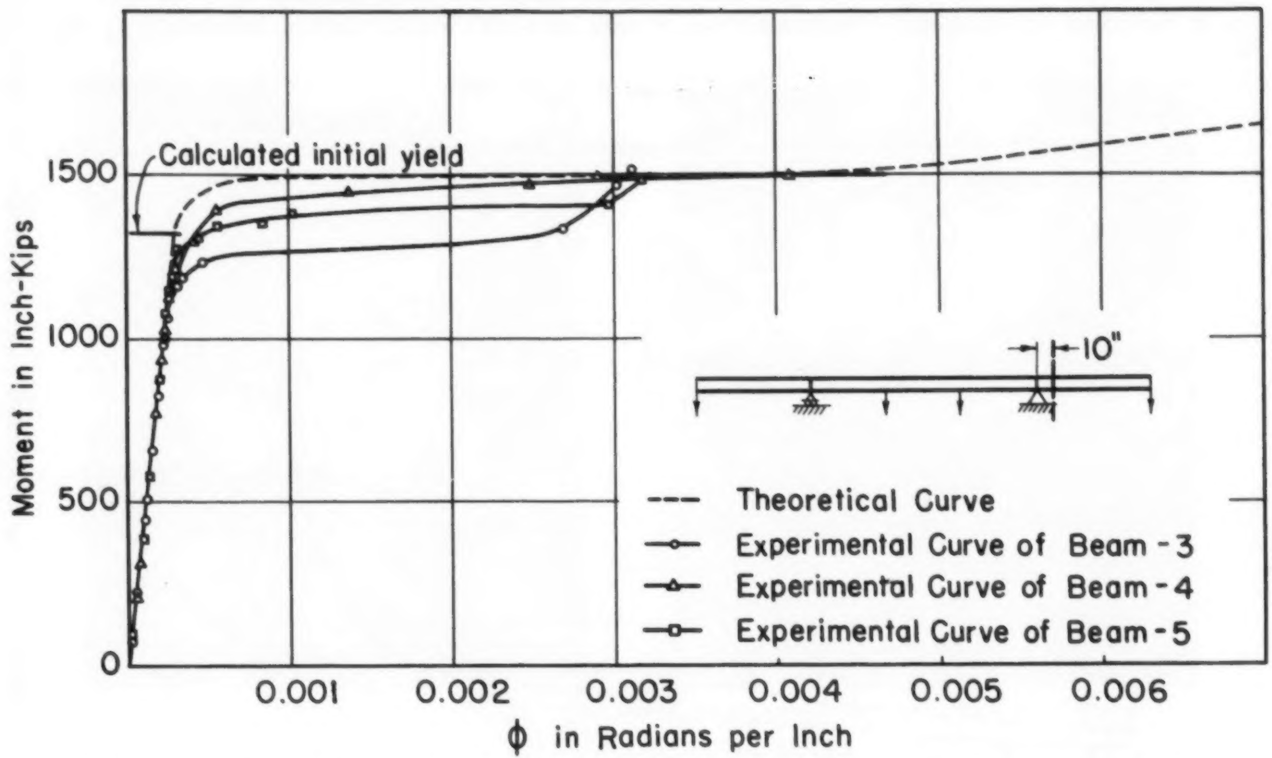


Fig. 17. Moment-Curvature Relationship of Continuous Beams of 8WF40 Shape at a Section Near a Support.

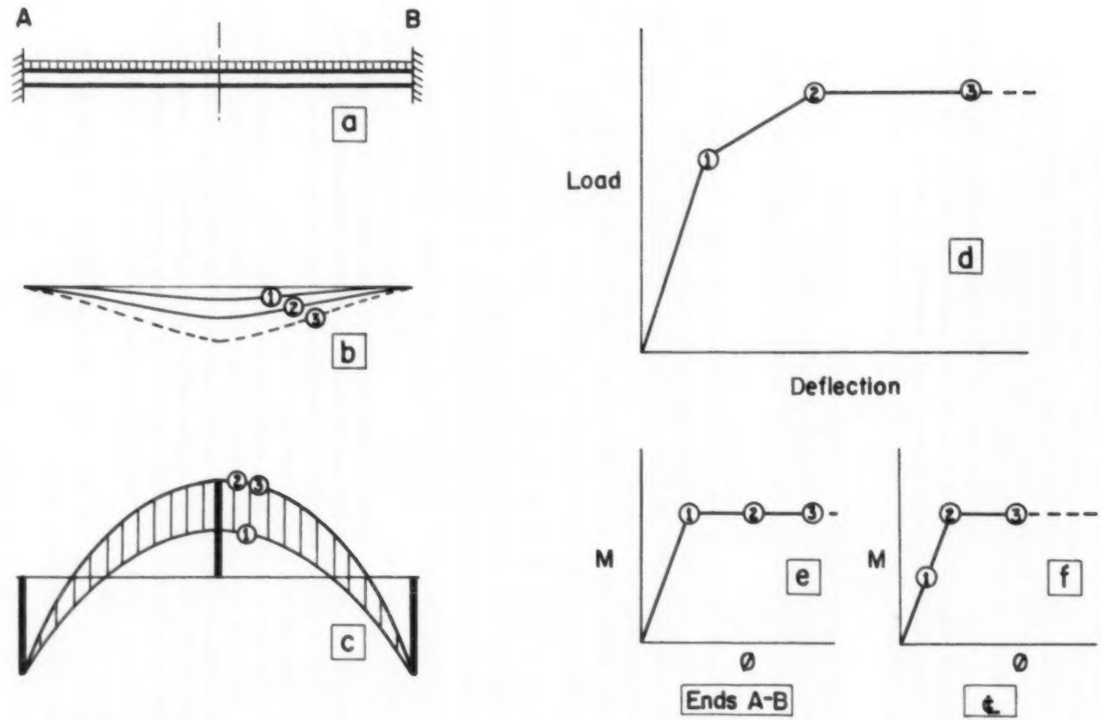


Fig. 18. The development of reserve strength through re-distribution of moment resulting from the formation of plastic hinges supports of a continuous beam.

5.3 Re-Distribution of Moment

Earlier it was mentioned that one of the sources of reserve strength is the development of full plastic yield of the cross-section (the "shape" factor). A second factor contributing to the reserve of strength—and usually to a much greater extent—is re-distribution of moment due to the formation of these hinges (the "redistribution" factor). It is here that the importance of the flat portion of the M-Ø curve comes into play. When the plastic moment is reached at a critical section, this moment is maintained at approximately constant value while the section rotates, re-distributing larger induced moments to other portions of the structure.

The fix-ended, uniformly-loaded beam of Fig. 18a will be used to illustrate how plastic hinges allow a structure to deform under load beyond the elastic limit, permit a re-distribution of moment and, thereby, an increase in load capacity. The numbers "1," "2," and "3" in Fig. 18 represent three phases of loading:

1) Attainment of first yield
2) First attainment of computed ultimate strength
3) An arbitrary deflection obtained by continued straining at the maximum load.

The idealized plastic hinge of Fig. 10 is assumed. In Fig. 18b are the deflection curves; in Fig. 18c, the moment diagrams; in Fig. 18d, the simplified load-vs-deflection curve; and in Figs. 18e and 18f, the M-Ø action at the ends and at the center, respectively.

By an elastic analysis, the moment diagram of Fig. 18c could be determined when yielding commences (Phase 1). The center moment is $\frac{pL^2}{24}$ and the end moment is $\frac{pL^2}{12}$. (p is the distributed load per unit length. L is the span length) On the load-vs-deflection curve of Fig. 18d, the load has reached Point 1. The moment-capacity has been used up at the ends (Fig. 18e); however, from Fig. 18f, since $M = \frac{1}{2}M_p$ at Phase 1, moment capacity is still available at the center of the beam. Therefore, as load increases beyond Phase 1, "hinge action" will start at the ends and the beam now behaves as if it were simply supported, except that the end moment remains constant at M_p. The deflection increases at a somewhat faster rate (the rate of increase being that of a simply-supported beam of length L).

At Phase 2 the beam reaches its maximum load since the moment capacity at the beam center is exhausted. Beyond Phase 2, the beam will continue to deform under constant load (Phase 3) in what has been termed a "mechanism."

The shaded portion of Fig. 18c represents the simple beam moment diagram that, due to re-distribution of moment, is superimposed upon the existing moment diagram (Phase 1) and corresponds to the increase of load between Phases 1 and 2 (Fig. 18d).

In the idealization of Fig. 10, $M_y = M_p$. By equilibrium, the load at first yield, P_y, may be determined from

$$\frac{P_y L}{8} = \frac{3}{2} M_y \qquad (2)$$

in which M_y is the computed yield moment. The ultimate load, P_u, may be computed from

$$\frac{P_u L}{8} = 2\ M_p \tag{3}$$

The ratio between the ultimate load and the yield load is

$$\frac{P_u}{P_y} = \frac{2\ M_p}{3/2\ M_y} = \frac{4}{3} \times \frac{M_p}{M_y} \tag{a}$$

As has been mentioned, the ratio $\frac{M_p}{M_y}$ is called the "shape factor," with an average value for WF shapes of 1.14. Therefore, using the more realistic M-Ø curve for rolled shapes (Fig. 11),

$$P_u/P_y = \frac{4}{3} \times 1.14 = 1.52 \tag{b}$$

In other words, the increase in load capacity of the beam beyond the load at initial yield is about 50%. The "shape factor" accounts for 14% and, in this example, the "re-distribution factor" accounts for 33 1/3% of the increase.

Plastic Analysis

The expression $P_u L/8 = 2\ M_p$, appearing above, is an equilibrium equation and constitutes the plastic analysis of a fix-ended beam uniformly loaded. Actually, conditions were considered that are similar to those in elastic analysis except that the process was simpler. Recently Baker and Horne have drawn a comparison between the basic conditions for each method.(2)

In elastic analysis one must consider three conditions:

1) Continuity - the deflected shape is examined and is the basis for continuity equations
2) Equilibrium - the load must be supported
3) Limiting Moment (or stress) - in elastic analysis the limiting moment is the yield moment.

In plastic analysis three similar conditions (or modifications thereof) must be considered. With regard to continuity, the situation is just the reverse: theoretically plastic hinges interrupt continuity, so the requirement is that sufficient plastic hinges form to allow the structure (or part of it) to deform as a mechanism. This could be termed a <u>mechanism</u> condition. The <u>equilibrium</u> condition is the same, namely, the load must be supported. Instead of initial yield, the limit of usefulness is the attainment of plastic hinge moments, not only at one cross-section but at each of the critical sections; this will be termed a <u>plastic moment</u> condition. The three conditions that must be considered in plastic analysis are, therefore,

1) Mechanism Condition
2) Equilibrium Condition
3) Plastic Moment Condition

and these are illustrated in Fig. 19 for a simple case.

To illustrate with the example shown in Fig. 18, three plastic hinges are necessary before the structure will deform as a mechanism. The corresponding maximum load is determined from the equilibrium condition (Eq. 3). Since the maximum moment is nowhere greater than the plastic moment, M_p, then the answer is the correct one.

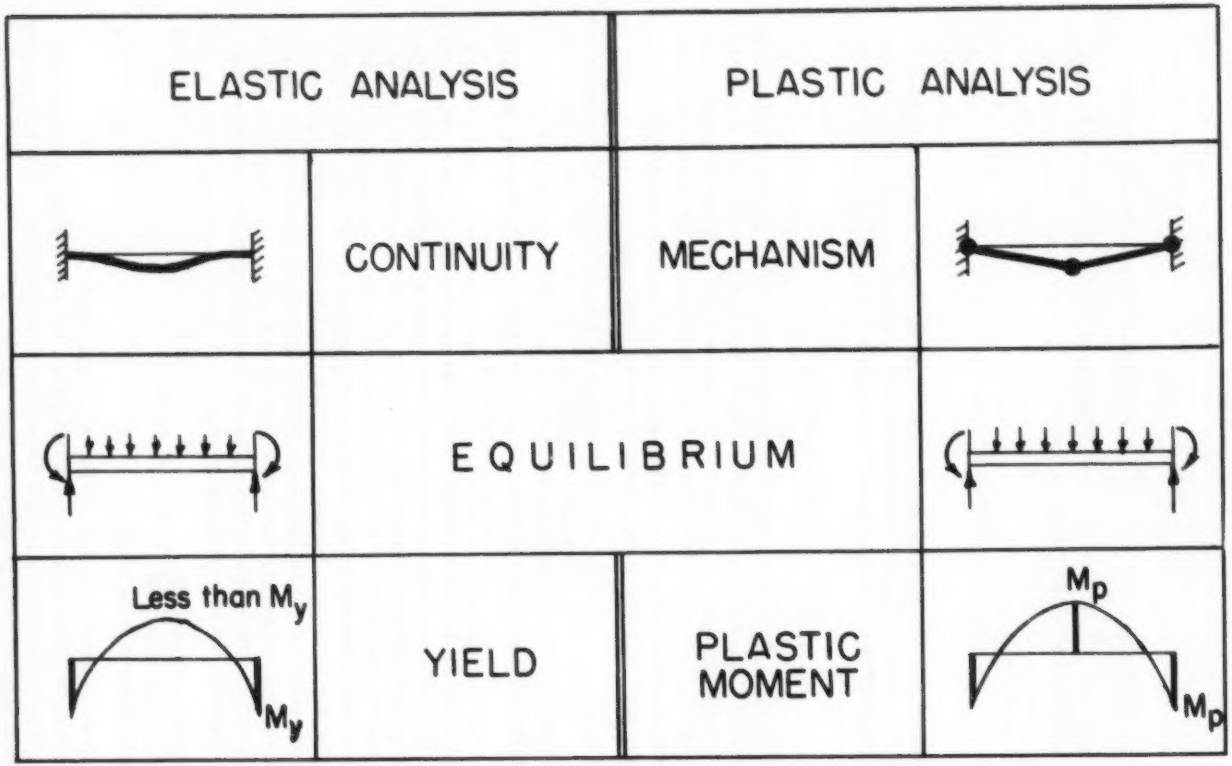
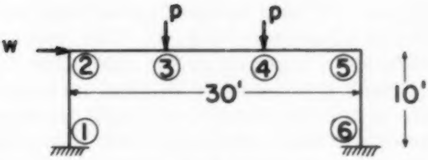
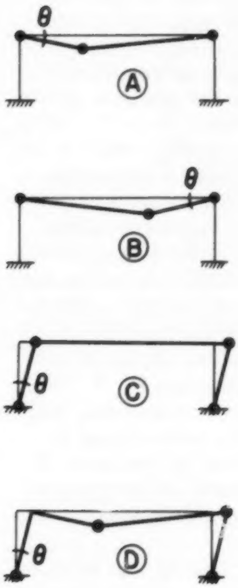


Fig. 19. Comparison between the conditions necessary for analysis of indeterminate structures by elastic (left) and plastic (right) methods.



LOADING

a



$$P\frac{L}{3}\theta + \frac{1}{2}P\frac{L}{3}\theta = M_p\left(\theta + \frac{3}{2}\theta + \frac{1}{2}\theta\right)$$

$$\boxed{P_u = 6\frac{M_p}{L}}$$

$$\frac{1}{2}P\frac{L}{3}\theta + P\frac{L}{3}\theta = M_p\left(\frac{1}{2}\theta + \frac{3}{2}\theta + \theta\right)$$

$$P_u = 6\frac{M_p}{L}$$

$$\frac{1}{9}P\frac{L}{3}\theta = 4M_p\theta$$

$$P_u = 108\frac{M_p}{L}$$

$$\frac{1}{9}P\frac{L}{3}\theta + P\frac{L}{3}\theta + \frac{1}{2}P\frac{L}{3}\theta = M_p\left(\theta + \frac{3}{2}\theta + \frac{3}{2}\theta + \theta\right)$$

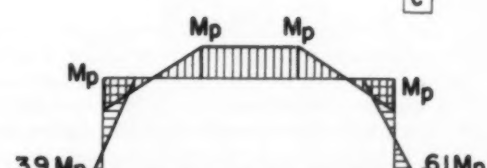
$$P_u = 9.3\frac{M_p}{L}$$

MECHANISMS

b

VIRTUAL WORK EQUATIONS

c



Equilibrium Moment Diagram

d

Fig. 20. Illustration of Plastic Analysis. According to assumed loading (a) with W = P/9, to each possible mechanism (b) there corresponds an equilibrium load determined by virtual work (c). Check obtained by constructing equilibrium moment diagram (d).

These various conditions will next be considered in an analysis of a frame that was tested at the Fritz Laboratory (Fig. 3). The frame with a 10 ft. column height and a 30-ft. span is shown in Fig. 20a. The section (12WF36) is uniform throughout. Loading consists of vertical load concentrated at the third points (to simulate uniformly-distributed load); the side load W represents the action of wind on the side of the structure.* For the particular proportions of the frame, W was taken as 1/9 P. This fixed-base portal frame is redundant to the third degree.

The method consists of examining the structure to see the different ways in which mechanisms can form by the development of plastic hinges (mechanism condition). To each possible mechanism there corresponds an ultimate load and this may quickly be determined either by a semi-graphical equilibrium method,(5) by use of the principle of virtual displacements,(10) or by other methods. (The condition being satisfied is the equilibrium condition.) It has been shown that the actual or correct mechanism for the problem is the one that gives the smallest value of the ultimate load.(6)

Possible plastic mechanisms for the problem are shown in Fig. 20b. These were not arrived at by guess but their selection followed a systematic procedure that is equally effective for more complicated structures than the one shown.(10,11) Mechanisms A and B are beam mechanisms; conceivably "local failure" could occur in the girders by the formation of the indicated hinges. Mechanism C is a panel mechanism and involves failure of the frame by sway to the right. Mechanisms A, B, and C are called elementary mechanisms, whereas Mechanism D results from a combination of mechanisms A and C. It will be noted that one of the hinges present in the two elementary mechanisms does not appear in the combined mechanism (Section 2).

For each of the Mechanisms A through D there exists a corresponding full plastic load, P_p. The principle of virtual displacements has been chosen to obtain this load. Using this principle, the external work done by the loads as the mechanism moves through a small displacement is equated to the internal work absorbed at each hinge as it rotates through a corresponding small angle. The resulting virtual work equations are shown in Fig. 20c.

Take Mechanism A, for example. As the structure deforms in this mechanism, the load P at the left moves through a distance, δ , that is equal to the small angle θ times the distance L/3; the load at the right moves through half the distance. Therefore, the external work is given by

$$W_e = \frac{PL\theta}{3} + \frac{1/2\ PL\theta}{3} \qquad \text{(c)}$$

During this motion work is done at the hinges. At Section 2 the internal work is equal to $M_p\theta$; at Section 5 it is equal to one-half this value; at Section 3 it is equal to $M_p\ (\theta + \frac{1}{2}\ \theta)$. Therefore the total internal work during the mechanism motion is

$$W_1 = M_p\ (\theta + 3/2\ \theta + 1/2\ \theta) \qquad (4)$$

* Rather than distribute a total load of 2W along the column height, the same action of the frame will be obtained in this problem if half the load is concentrated at the top of the column as load W.

Equating the two (for equilibrium) gives

$$P_u = 6M_p/L \qquad (5)$$

The lowest load is obtained in the case of Mechanism A (or Mechanism B which is identical). The ultimate load on the structure is therefore

$$P_u = 6M_p/L \qquad \text{(d)}$$

As a check that the proper mechanism has been selected, the third (or plastic moment) condition is considered—at no point may the moment be greater than M_p. This check consists of computing by equilibrium the moments at each critical section in the frame and constructing the moment diagram. If at each point of the frame the moment is no greater than M_p, then the answer is the correct one. The bending moment diagram for the frame is shown in Fig. 20d for the case where the side load W was equal to 1/9 of the concentrated vertical load P.

Now, suppose the problem was to find how much side load the frame would stand while the full magnitude of the vertical load is maintained on the structure. This situation might arise in a consideration of the blast resistance of frames. There are two mechanisms by which the frame can fail under side load. These are Mechanisms C and D and the procedure would be to try each. The virtual work equation for Mechanism D written in the terms of the unknown W, is

$$\frac{WL}{3}\,\theta + \frac{PL\theta}{2} = 5\ M_p\ \theta \qquad (6)$$

Solving this for W (using a value of $P = 6\ M_p/L$), the allowable side load equals $6\ M_p/L$.

Now, for Mechanism C the virtual work equation in terms of the unknown W is given by

$$W\,\frac{L}{3}\,\theta = 4\ M_p\ \theta \qquad (7)$$

or $W = 12\ M_p/L$. Therefore the correct maximum side load W—the lowest value—is equal to $6\ M_p/L$ or nine times the normal wind load.

How well does the test of a full-size frame bear out the theory? In Fig. 21 are shown two load-vs-deflection curves obtained from the test shown pictorially in Fig. 3. The one on the left is for the first test on the structure in which the side load was equal to 1/9 of the vertical load, representing the practical condition treated in Fig. 20. The frame carried the computed ultimate load. The test was then stopped and the second phase completed.

The resulting curve of horizontal load vs horizontal deflection as the side load was increased to a maximum value is shown to the right. The frame did not quite reach the value computed according to the Eq. 6, but this is to be expected in view of the plastic deformations to which the frame had been subjected in the first test.

This figure, therefore, shows that the predictions of plastic analysis are confirmed by actual test.

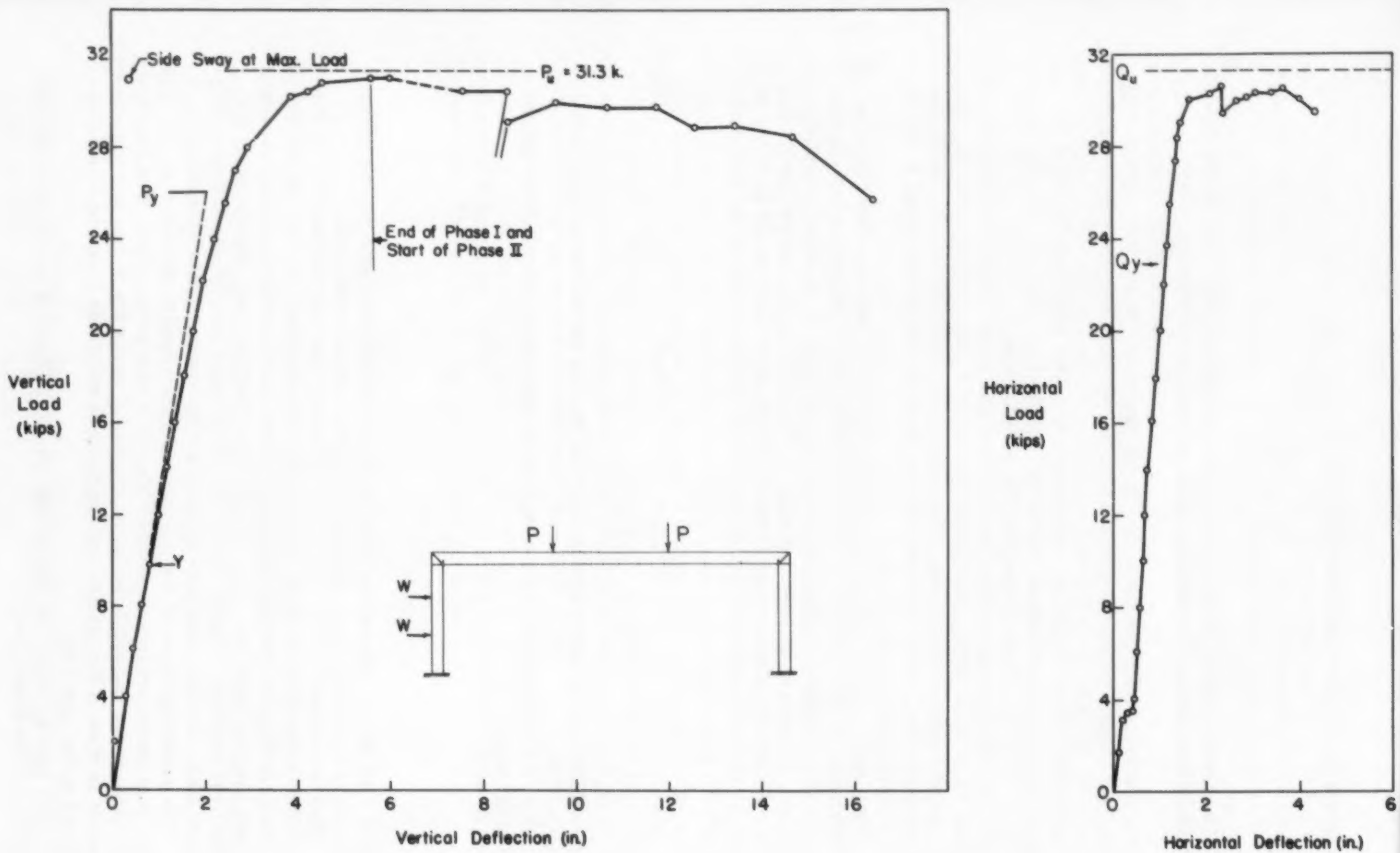


Fig. 21. Load-vs-Deflection Curves for Frame Test (T4).

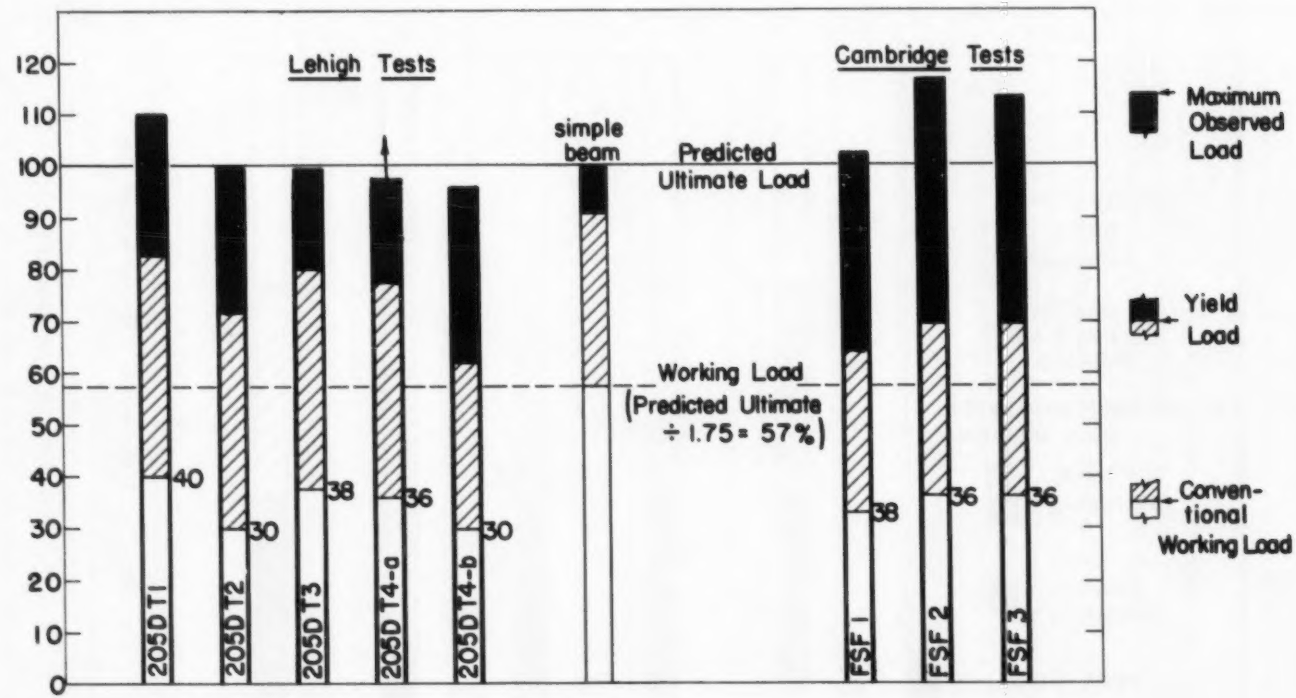


Fig. 22. Strength of Portal Frames. Each bar graph represents results of a "full-size" frame test. Simple beam graph shown for comparison.

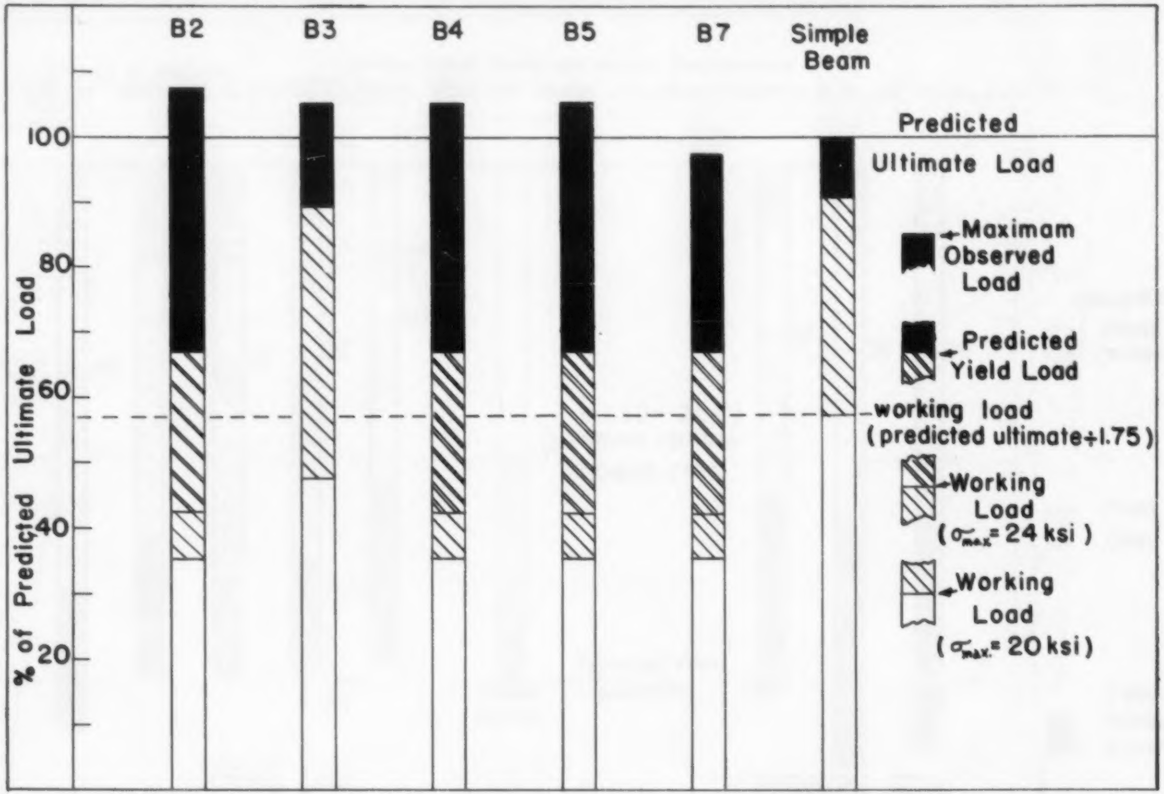


Fig. 23. The Strength of Continuous Beams. Each bar chart represents results of a "full-size" continuous beam test.

Fig. 22 shows the ultimate strength of single span rigid frames. Each of the bar charts represents actual values from a series of frame tests made either at Lehigh(9) or at the University of Cambridge in England under Professor Baker's direction.(3)

The vertical scale is the percent of predicted ultimate load. Thus a test that reached 100 percent on the vertical scale reached the load that is the basis of plastic analysis. The dotted line across the figure is a possible plastic design working load.

The top of the open portion of each bar represents the allowable conventional working load on a percentage-of-ultimate-load basis. The top of the cross-hatched portion indicates the predicted yield load. The solid portion up to the top of each bar represents the reserve of strength above the elastic limit and up to the maximum strength observed in the test. The predicted yield and ultimate loads are computed on the basis of measured material properties.

The case for plastic design is illustrated by the following observations in connection with Fig. 22:

1) The reserve in strength above present conventional working loads is considerable in continuous steel structures (150 to 200%).
2) The reserve of strength above the nominal theoretical yield point is quite large in every case. In some instances as much capacity is disregarded as is used in conventional design. (Compare black with white portions of bars.)
3) The plastic strength may be predicted quite closely.
4) A possible working load for plastic design in indicated by the dotted line. The increase in working load, compared with conventional load is usually greater than 30%. (In these tests it ranged from 42% to 90%).
5) Stresses at the plastic design working load are below the computed elastic limit.
6) Since the allowable load on the simple beam (shown in the figure) is the same for both elastic and plastic design, use of ultimate strength as the design criterion provides at least the same margin of reserve strength as is presently afforded in the conventional design of simple beams.

Fig. 23 shows similar information, but for continuous beams. Also shown here, percentagewise, are the conventional working loads permitted by the AISC provision increasing the allowable stress to 24,000 psi at the interior supports of continuous beams. Note that these loads are substantially less than would be permitted using ultimate strength design with a load factor of safety of 1.75.

The general observations made above concerning Fig. 22, are equally applicable to Fig. 23.

SUMMARY

The aim of this report has been to document the applicability of plastic analysis to structural design through an examination of test results correlated with applicable theory and in comparison with the provisions of present specifications. The following is presented in summary:

1) The unique feature of plastic design is that the ultimate load, rather than the yield stress, is regarded as the design criterion.

2) The justification for plastic design, in brief, is safety and economy resulting from balanced design. Substantial cost savings are in sight through the more economical use of steel and a saving of time in the design office brought about by the use of the simpler plastic methods. At the same time, building frames are provided which are more logically designed for greater over-all strength.
3) Tests of continuous beams and portal frames in a size approaching full scale illustrate that the reserve strength of steel is considerable and that it can be predicted with reasonable certainty. (Fig. 2, 3, and 8).
4) Two factors account for the development of reserve strength in continuous structures. One is the "shape factor," representing the theoretical increase in moment strength of a section bent beyond the yield point; the ratio of full moment strength (M_p) to computed yield moment (M_y) averages about 1.14 (Fig. 11). The other factor is the "re-distribution factor;" when the plastic moment, M_p, is reached at a critical section, the moment is maintained at approximately constant value while the section rotates, re-distributing larger induced moments to other portions of the structure. (Fig. 18 has been used to describe this action). Percentagewise, the latter factor is more important than the former.
5) Basic to the actual development of computed ultimate load is the proper formation of the all-important "plastic hinges." Tests show that properly proportioned components (connections, columns, beams) can be depended upon to develop the plastic hinge moment and to have adequate reserve of curvature. (Fig. 16).
6) In the process of analysing a structure to determine the maximum plastic strength, it is necessary to consider three conditions: (1) a Mechanism condition, (2) an Equilibrium condition, and (3) a Plastic Moment condition. These have direct parallels in elastic analysis (Fig. 19). Based on these conditions, an indeterminate steel frame may be analysed by a systematic procedure that is usually more rapid than the corresponding elastic analysis of the same structure.
7) The case for plastic design is illustrated by comparing the maximum strength of test frames with the computed ultimate load, with the computed first yield, and with the allowable load according to conventional design. (Figs. 22, 23). According to plastic design, the increase in working load, compared with present allowable loads, is usually greater than 30%. At the same time, stresses at the "plastic design" working load are below the computed elastic limit. Further, use of ultimate strength as the design criterion provides at least the same margin of reserve strength as is presently afforded in the conventional design of simple beams.

ACKNOWLEDGMENTS

This report has been prepared as a result of research carried out at Lehigh University in the Fritz Engineering Laboratory. Wm. J. Eney is Director of the Laboratory and Head of the Department of Civil Engineering and Mechanics. The work was done as part of a program on Welded Continuous Frames and Their Components being supervised by the Lehigh Project Subcommittee of the Structural Steel Committee, Welding Research Council. Sponsors of the program are: American Institute of Steel Construction, American Iron and Steel Institute, Column Research Council (Advisory),

Navy Department (Contract 39303)—Office of Naval Research, Bureau of Ships, Bureau of Yards and Docks, Welding Research Council. Sincere appreciation is expressed to T. R. Higgins, Chairman of the committee, and to the members for valuable advice received during the course of the investigation.

This program was first started at Lehigh University by Bruce G. Johnston, now at University of Michigan. Active in supervising various aspects of the study have been K. E. Knudsen, Bruno Thurlimann, F. W. Schutz, and Robert L. Ketter. The valuable contributions of these and of the numerous Research Assistants who have participated in the program are gratefully acknowledged. The assistance furnished by members of the Fritz Laboratory staff in the preparation of this manuscript is gratefully acknowledged.

REFERENCES

1. Baker, J. F., "Shortcomings of Structural Analysis," Trans. N. E. Coast Inst. Eng. Ship., Vol. 68, p. 31, 1951.

2. Baker, J. F. and Horne, M. R., "New Methods in the Analysis and Design of Structures in the Plastic Range," British Welding Journal 1 (7) page 307, July, 1954.

3. Baker, J. F. and Roderick, J. W., "Tests on Full-Scale Portal Frames," Inst. C. E. (Br.) p. 71, January 1952.

4. Beedle, L. S. and Johnston, B. G., "Rules of Practice in Plastic Design," Interim Report No. 26, Fritz Laboratory Report No. 205.20, Lehigh University, August, 1954.

5. British Construction Steelwork Association, "The Collapse Method of Structural Design," BCSA Publication No. 5, (Br.), 1952.

6. Greenberg, H. J. and Prager, W., "Limit Design of Beams and Frames," Trans. ASCE, 117, p. 447, 1952.

7. Hoff, N. J., Discussion of Ref. 8, Welding Journal, 33(1), p. 14-s, January, 1954.

8. Johnston, B. G., Yang, C. H., and Beedle, L. S., "An Evaluation of Plastic Analysis as Applied to Structural Design," Welding Journal 32 (5), p. 224-s, May, 1953.

9. Schilling, C. G., Schutz, F. W., and Beedle, L. S., "Behavior of Welded Single-Span Frames Under Combined Loading," F. L. Report No. 205D., March, 1955, to be published in Welding Journal.

10. Symonds, P. S. and Neal, B. G., "Recent Progress in the Plastic Methods of Structural Analysis," Journal of the Franklin Institute, 252 (5) page 383, November 1951, and 252 (6) page 469, December, 1951.

11. Thurlimann, B. T., "Analysis of Frames for Ultimate Strength," Fritz Laboratory Report No. 205.29, May 1955.

12. Yang, C. H., Beedle, L. S., and Johnston, B. G., "Residual Stress and the Yield Strength of Steel Beams," Welding Journal, 31(4), p. 205s, April, 1952.

NOMENCLATURE

b = Flange width

d = Depth of section

E = Young's modulus of elasticity

f = Shape factor = $\frac{M_p}{M_y} = \frac{Z}{S}$

L = Span length

M = Moment

M_p = Plastic moment

M_y = Moment at which yield point is reached in flexure

P = Concentrated Load

p = Distributed load per unit of length

P_u = Ultimate load on a structure computed by simple plastic theory.

P_y = Load at computed initial yield point

S = Section modulus, $\frac{I}{c}$

t = Flange thickness

W = Side load

w = Web thickness

Z = Plastic modulus, $Z = \frac{M_p}{\sigma_y}$

δ = Deflection

δ_y = Deflection at computed yield point

θ = An arbitrary small angle

σ_y = Yield stress level

ϕ = Rotation per unit length, or average unit rotation; curvature